# Teacher notes
# Topic A

## An instructive momentum problem

A ball is inside a box of length $L$ that rests on a frictionless floor. Initially the ball and the box are at rest. The ball is given a push and moves towards the right-hand end of the box as shown. The speed of the ball **relative to the box** is $u$.

The mass of the box is equal to the mass of the ball.

(a) Explain why the box will move to the left.
(b) Calculate, in terms of $u$, the velocity of the box.
(c) Calculate the distance travelled by the ball and that travelled by the box by the time the ball reaches the other end of the box.
(d) What happens when the ball reaches the end of the box?

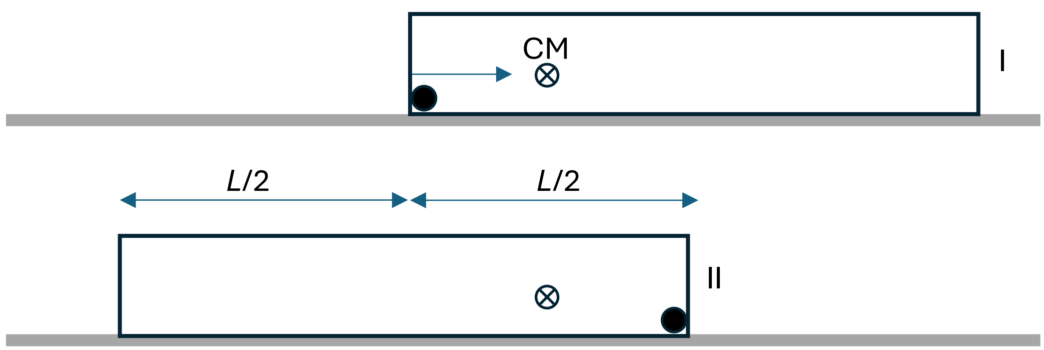
## Answers

(a) There is no external force acting on the system of box & ball. Hence the center of mass must remain in the same place. This can happen only if the box moves to the left. This can also be answered in terms of momentum conservation, see (b).

(b) Initially the momentum of the system is zero and hence will stay zero. Let the speed of the box be $v$ with respect to the ground. The velocity of the ball relative to the ground is $u-v$. Conservation of momentum then says that $M(u-v)-Mv=0$. Solving we find $v=\frac{u}{2}$ for the speed and so $-\frac{u}{2}$ for the velocity.

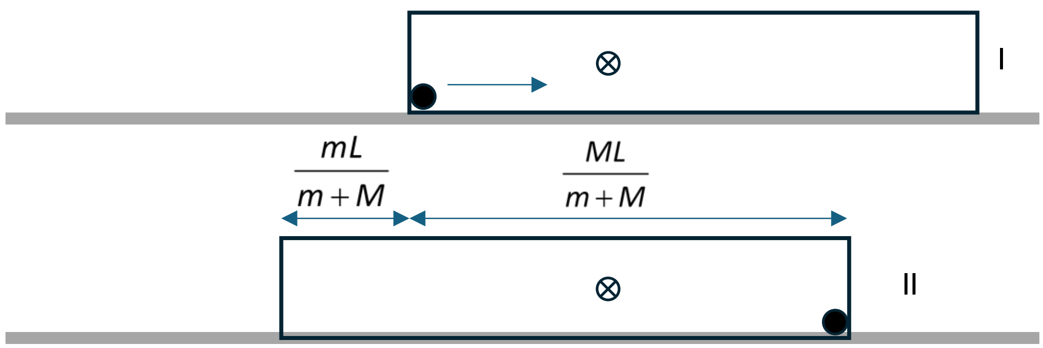
(c) The time taken for the ball to reach the other end of the box is $t=\frac{L}{u}$. So, with respect to the ground the ball will move a distance $x_{\text{ball}}=(u-v)t=(u-\frac{u}{2})\times\frac{L}{u}=\frac{L}{2}$.

The box will move to the left a distance $x_{\text{box}}=vt=\frac{u}{2}\times\frac{L}{u}=\frac{L}{2}$.




(d) If the ball collides elastically with the wall, it will turn back with the same speed and the box will start moving to the right, i.e. the box will be oscillating between the two positions, I and II. If, however, the ball gets stuck to the wall then the box will shop moving. If it takes a time $\Delta t$ for the ball to be brought to rest, the impulse delivered by the box on to the ball is $M\frac{u}{2}$. The opposite impulse is delivered to the box by the ball which is just the right amount to bring the box to rest.

It is instructive to try this for different ball and box masses. Let *m* be the ball mass and *M* that of the box. Then, $m(u-v)-Mv=0$. Solving we find $v=\dfrac{mu}{m+M}$. The distance travelled by the ball is then $x_{\text{ball}}=(u-v)t=(u-\dfrac{mu}{m+M})\times\dfrac{L}{u}=\dfrac{M}{m+M}L$ and that of the box is $x_{\text{box}}=vt=\dfrac{mu}{m+M}\times\dfrac{L}{u}=\dfrac{m}{m+M}L$. For *M* > *m* the diagram would be something like:




Notice that

$$x_{\text{box}}+x_{\text{ball}}=\frac{mL}{m+M}+\frac{ML}{m+M}=L$$

as it should be.